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KINEMATICS OF LAYING AN AUTOMATED WEAPON SYSTEM

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July 2017

U.S. ARMY ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER

Weapons and Software Engineering Center

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14. ABSTRACT
This report describes the unique issues of pointing an automated weapon by an indirect fire system. Due to different frames of reference, between the calculated firing solution and the platform that the weapon is mounted to, a mathematical transformation is required to move the firing solution from its reference frame to a reference frame that is meaningful to the weapon system. This report describes the aforementioned problem in detail and provides a systematic solution using a series of equations.

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INTRODUCTION

In artillery, the process of aiming a weapon is referred to as “gun laying.” Gun laying is achieved by performing movements that align the axis of the gun barrel with the calculated lay angles. The gun angles are specified relative to a horizontal plane and a vertical plane. A gun is traversed in the horizontal plane, and elevated in the vertical plane, to range it to its target. The traverse and elevation values make up the aiming portion of the “firing solution.”

Reference Frames

The firing solution provides values referenced to an orthogonal earth reference frame. The origin of the frame is placed at the weapon’s location on the earth’s surface. From the origin, north is the X axis, east is the Y axis, and the Z axis points downward toward the center of the earth’s sphere as shown in figure 1.

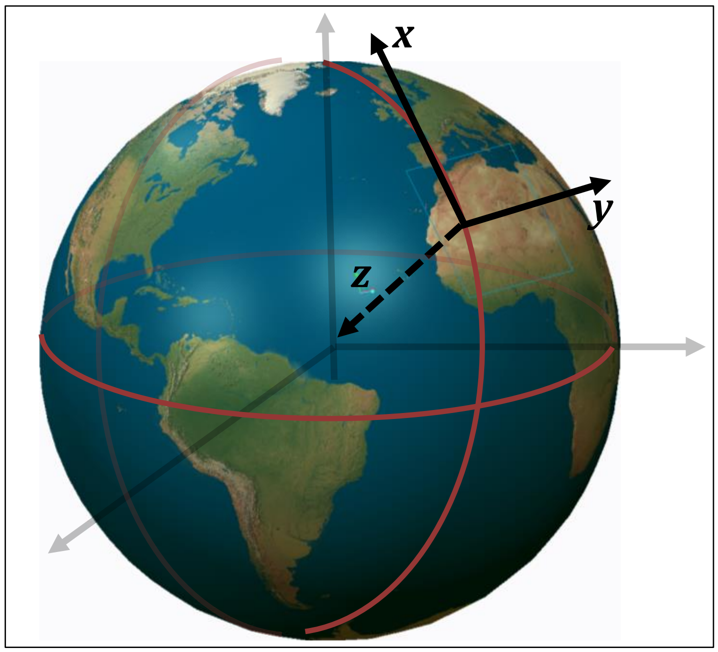

Figure 1
Earth reference frame

In the case of an automated weapon, actuators traverse and elevate the gun barrel relative to the platform the barrel is mounted to. The gun to platform relationship is defined using a reference frame that is different than the earth reference frame described previously. The origin of the platform reference frame exists at the intersection of the traverse axis and the elevation axis. In some instances, the elevation axis of rotation may not intersect with the traverse axis. However, since translation is not a concern for pointing, this condition can be ignored and the axes can be assumed to have a common origin, as long as they exist within a common plane. For the platform reference frame, the Z axis extends down along the traverse axis of rotation. The X axis extends toward what is established to be the 0 reference of the platform at a right angle to the Z axis (typically what would be considered the front of the platform). The Y axis completes the orthogonal reference frame using the right hand rule. An example platform reference frame is illustrated in figure 2.

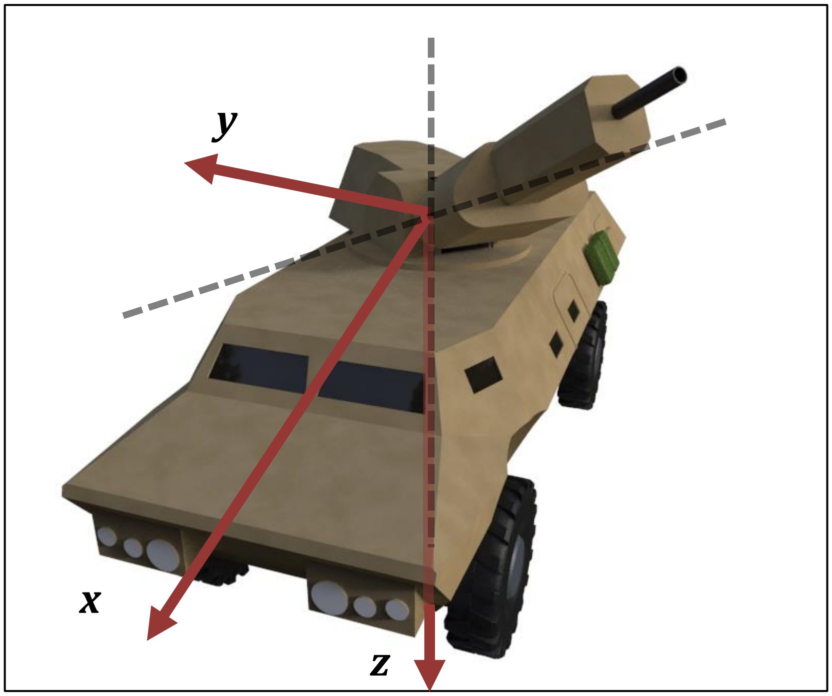

Figure 2
Platform reference frame

When performing rotation transformations, an intermediate reference frame is used to perform the operations. This is referred to as the local reference frame. The local reference frame is initially coincident with the starting reference frame. It is subsequently rotated about its axes in a series of motions that result with it being coincident with the final reference frame. In the case of laying a weapon, the centerline of the gun tube is represented by the X axis of the local reference frame.

The Problem

Since the platform reference frame is typically not coincident with the earth reference frame, a disparity exists between the feedback used to control motion of the gun and the feedback indicating current attitude of the gun relative to earth. To resolve this issue, the firing solution must be translated from the earth reference frame to the platform reference frame. The actuator control system can then use that information to command traverse and elevation motions required to lay the gun on the intended target.

METHODS, ASSUMPTIONS, AND PROCEDURES

Conventions and Variable Definitions

Before describing the formulas that convert the firing solution to the correct reference frame, the conventions and variables to be used are defined in table 1, which defines the variables used in the subsequent calculations:

Table 1
Variable definitions

Variable	Definition
X_e, Y_e, Z_e	Axis of earth reference frame
X_p, Y_p, Z_p	Axis of platform reference frame
ψ_{Fe}	Firing solution yaw rotation (earth reference)
θ_{Fe}	Firing solution pitch rotation (earth reference)
ψ_{Ge}	Gun attitude yaw rotation (earth reference)
θ_{Ge}	Gun attitude pitch rotation (earth reference)
φ_{Ge}	Gun attitude roll rotation (earth reference)
x_{Fe}, y_{Fe}, z_{Fe}	Firing solution cartesian coordinates (earth reference)
ψ_{Gp}	Gun attitude yaw rotation (platform reference)
θ_{Gp}	Gun attitude pitch rotation (platform reference)
ψ_{Fp}	Firing solution yaw rotation (platform reference)
θ_{Fp}	Firing solution pitch rotation (platform reference)
x_{Fp}, y_{Fp}, z_{Fp}	Firing solution Cartesian coordinates (platform reference)

A coordinate system uses one or more values, or coordinates, to uniquely determine the position of a point or other geometric element. The calculations that solve the problems in this report require that unit vectors be represented in two different types of coordinate systems: Cartesian and spherical. Equivalent Cartesian and spherical coordinate systems are shown in figures 3 and 4.

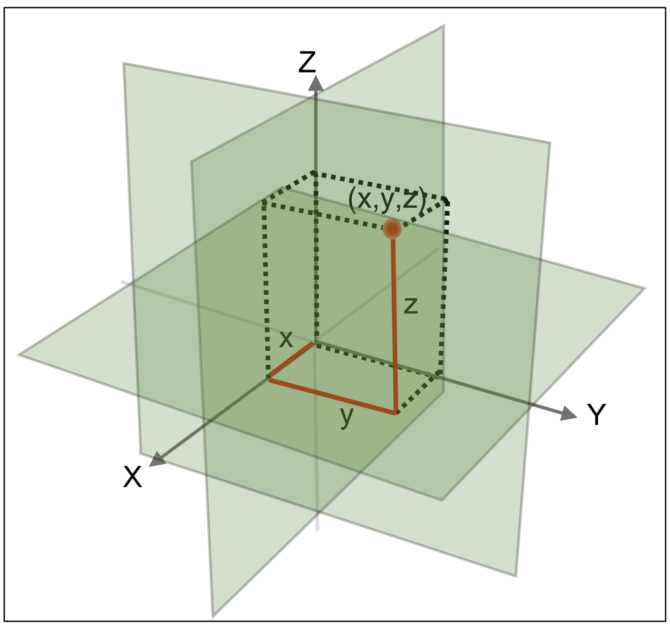

Figure 3
Cartesian coordinate system

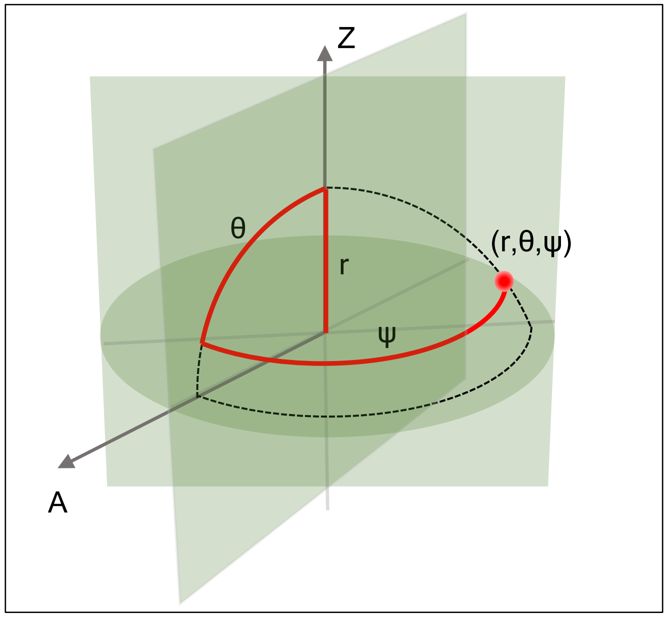

Figure 4
Spherical coordinate system

When describing rotations, the Greek letters ψ, θ, and φ are used to describe angular rotation quantities of yaw, pitch, and roll about the Z, Y, and X axes respectively. By convention, positive rotation is defined as clockwise when looking outward from the origin of the frame (fig. 5).

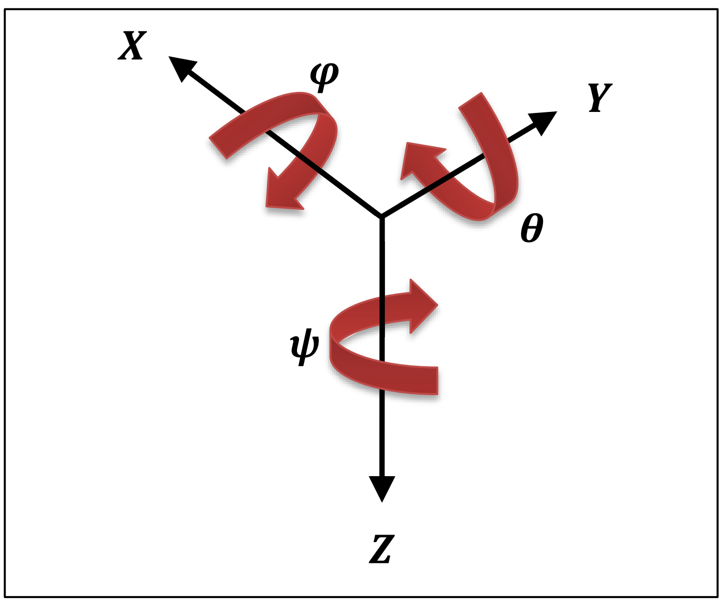

Figure 5
Rotation symbols and directions

The attitude of an object relative to a fixed reference frame is defined by means of a series of rotations about the object's axes starting with the objects local reference frame aligned with the fixed reference frame. The order of these rotations is ψ about Z, θ about Y, then φ about X. Figure 6 illustrates this concept.

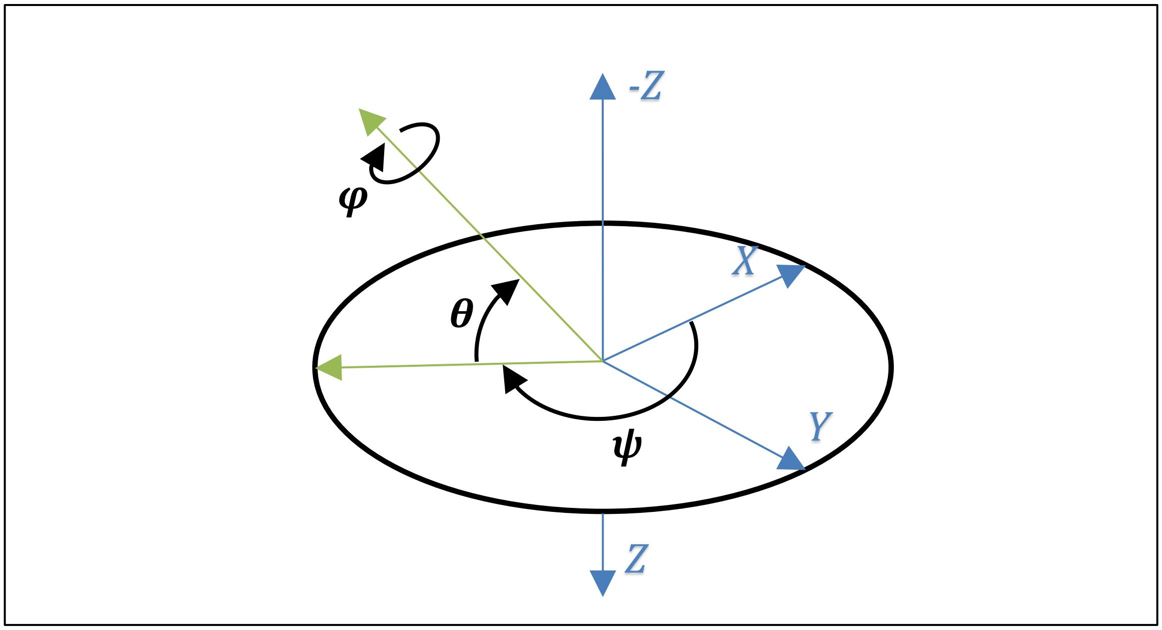

Figure 6
Attitude description

Rotation Matrices

A rotation matrix is used to perform a rotation in Euclidean space. When working in three dimensions, there are three basic rotation matrices that are used to rotate vectors about the X, Y, and Z axes.

$$R_Z(\psi) = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$R_Y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \tag{2}$$

$$R_X(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix} \tag{3}$$

Transformation of a Vector

If two vectors, p and p′, both describe the Cartesian coordinates of a point P using different reference frames, a rotation matrix can be used to represent the transformation of the vector coordinates from one reference frame to the other. The rotation matrix R represents the transformation matrix of the vector coordinates in frame O- X′ Y′ Z′ into the coordinates of the same vector in frame O-XYZ.

$$p = Rp' \tag{4}$$

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Since R is an orthogonal matrix (i.e., $R^TR=I_3$), it follows that p′ can be calculated using the transpose of R:

$$p' = R^T p \tag{5}$$

Conversion Between Cartesian and Spherical Coordinate Systems

At times, it is necessary to convert from a Cartesian coordinate system to a spherical coordinate system. These calculations assume that the Cartesian X axis aligns with the spherical A axis and that the Z axes align. Since unit vectors are used, the actual radius r is irrelevant and set to 1. These equations are used for conversion.

$$x = \cos(\psi)\cos(\theta) \tag{6}$$

$$y = \sin(\psi)\cos(\theta) \tag{7}$$

$$z = -\sin(\theta) \tag{8}$$

$$\psi = \operatorname{atan2}(\frac{y}{x}) \tag{9}$$

$$\theta = \operatorname{atan2}(\frac{-z}{\sqrt{x^2+y^2}}) \tag{10}$$

Note that atan2 is a programming function that uses the input to determine the quadrant and returns the corrected angle from an arctangent operation.

Transformation of Earth Referenced Lay to Platform Reference Frame

Given the yaw and pitch values that form a firing solution in the earth reference frame, the first step in transforming the solution to the platform reference frame is to create a vector in Cartesian coordinates using equations 6 through 8.

$$x_{\mathrm{Fe}} = \cos(\psi_{\mathrm{Fe}})\cos(\theta_{\mathrm{Fe}}) \tag{11}$$

$$y_{\mathrm{Fe}} = \sin(\psi_{\mathrm{Fe}})\cos(\theta_{\mathrm{Fe}}) \tag{12}$$

$$z_{\mathrm{Fe}} = -\sin(\theta_{\mathrm{Fe}}) \tag{13}$$

To transform the Cartesian coordinates from the earth reference frame to the platform reference frame, a rotation matrix is defined as the product of a series of rotations via the basic rotation matrices defined in equations 1 through 3. Starting with the local reference frame, coincident with the earth reference frame, a rotation of ψ_{Ge} is performed about Z, then θ_{Ge} is rotated about Y, and finally φ_{Ge} is rotated about X. These yaw, pitch, and roll rotations result in the reference frame being coincident with the current gun tube axis. The local reference frame is then rotated by the negative of θ_{Gv} about Y and the negative of ψ_{Gv} about Z. These motions result in the reference frame being coincident with the platform reference frame. These five series of rotations are depicted in equation 14.

$$R = R_Z(\psi_{\mathrm{Ge}})R_Y(\theta_{\mathrm{Ge}})R_X(\varphi_{\mathrm{Ge}})R_Y(-\theta_{\mathrm{Gp}})R_Z(-\psi_{\mathrm{Gp}}) \tag{14}$$

Substituting the functions with the rotation matrix equations found in equations 1 through 3 yields:

$$R = \begin{bmatrix} c\psi_{\mathrm{Ge}} & -s\psi_{\mathrm{Ge}} & 0 \\ s\psi_{\mathrm{Ge}} & c\psi_{\mathrm{Ge}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_{\mathrm{Ge}} & 0 & s\theta_{\mathrm{Ge}} \\ 0 & 1 & 0 \\ -s\theta_{\mathrm{Ge}} & 0 & c\theta_{\mathrm{Ge}} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\varphi_{\mathrm{Ge}} & -s\varphi_{\mathrm{Ge}} \\ 0 & s\varphi_{\mathrm{Ge}} & c\varphi_{\mathrm{Ge}} \end{bmatrix} \begin{bmatrix} c(-\theta_{\mathrm{Gp}}) & 0 & s(-\theta_{\mathrm{Gp}}) \\ 0 & 1 & 0 \\ -s(-\theta_{\mathrm{Gp}}) & 0 & c(-\theta_{\mathrm{Gp}}) \end{bmatrix} \begin{bmatrix} c(-\psi_{\mathrm{Gp}}) & -s(-\psi_{\mathrm{Gp}}) & 0 \\ s(-\psi_{\mathrm{Gp}}) & c(-\psi_{\mathrm{Gp}}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

Multiplication of the transpose of the rotation matrix by the firing solution vector results in the firing solution vector in terms of the platform reference frame.

$$\begin{bmatrix} x_{\mathrm{Fp}} \\ y_{\mathrm{Fp}} \\ z_{\mathrm{Fp}} \end{bmatrix} = R^T \begin{bmatrix} x_{\mathrm{Fe}} \\ y_{\mathrm{Fe}} \\ z_{\mathrm{Fe}} \end{bmatrix} \tag{16}$$

The new firing solution vector is then converted back to spherical coordinates using equations 9 and 10.

$$\psi_{\mathrm{Fp}} = \mathrm{atan2}\left(\frac{y_{\mathrm{Fp}}}{x_{\mathrm{Fp}}}\right) \tag{17}$$

$$\theta_{\mathrm{Fp}} = \mathrm{atan2}\left(\frac{-z_{\mathrm{Fp}}}{\sqrt{x_{\mathrm{Fp}}{}^2 + y_{\mathrm{Fp}}{}^2}}\right) \tag{18}$$

RESULTS AND DISCUSSIONS

The equations and methods described in this report have been implemented on several systems with great success. The accuracy and resolution of the sensors and actuators determine how accurately the weapon can be pointed. The values needed to perform the previous calculations required the yaw, pitch, and roll of the gun barrel relative to earth and the yaw and pitch of the gun tube relative to the platform. The gun to earth angles are typically provided by an inertial navigation system (INS), and the gun to platform angles are typically provided by angular sensors on the azimuth and elevation axes.

There are some additional real world factors that need to be considered as well. The INU is mounted to a structure that moves in unison with the gun tube. However, the reference frame representing the INU may not perfectly align with that of the gun tube. To correct for this, the INU is boresighted to the gun tube. The boresighting process determines the misalignment values and stores three corrections in the INU representing yaw, pitch, and roll offsets. This allows the INU to take any misalignments between itself and the axis of the gun into consideration and adjust its output to represent the attitude of the gun itself.

Another consideration is ensuring that the earth referenced gun attitude output by the INU uses the same north reference as the reference used for the firing solution. There are three primary ways of referencing north. True north points to the northern axis of the earth's rotation. Magnetic north is aligned with the earth's magnetic field, which will differ from true north, based on location and also changes with time. Grid north is north as represented on a two dimensional map. Military

systems typically specify their location using the military grid reference system that breaks locations on the surface of the earth down into a set of grid zones. For a location within a grid zone, there could be an error between its grid north and true north, which is corrected using a declination correction. For instance, if the INU is outputting true north referenced values and the firing solution is referenced to grid north, a declination offset needs to be applied prior to performing the calculations.

Furthermore, since the platform could be a vehicle with a compliant suspension, the platform reference frame can change with respect to the earth reference frame as the weapon moves and the suspension is loaded in different areas. This becomes a control problem to which there are many approaches. Suffices to say that the final platform solution may differ from the solution calculated prior to moving.

CONCLUSIONS

Through the use of rotation transformations and trigonometric functions, solutions were found for the aforementioned problem. These calculations allow the system controlling the actuators to successfully point a boresighted weapon to the desired orientation regardless of initial platform orientation.

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